

A Five-Step Problem-Solving Process

The Definition of a Problem

Before we discuss problem solving, we first probe into the definition of a problem. Is the following a problem? Keefe has two cats and three dogs. How many pets does he have? For many pre-school-aged children, this could be considered a problem. For the average second-grade student, this problem would be solved within a matter of seconds—no problem. To truly be considered a problem, a mathematical quest must contain some effort or thought on the part of the solver (Brownell, 1942; Polya, 1945). As a teacher you must remember that what is a problem for some students may be a mere exercise for others.

Problem solving involves a variety of skills.

Problem-solving situations call upon children to retrieve previously learned information and apply it in new or varying situations. Knowing the basic arithmetic skills, knowing when to incorporate them into new contexts, and then being able to do so are three distinct skills. Having all three skills makes problem solving easier, but inability in one does not mean that a student does not understand a problem. It may mean that the student's learning style has not been addressed. Similarly, because students can carry out the operations in isolation does not mean they know when to apply them or how to interpret the numbers involved (Bley & Thornton, 2001, p. 37).

Good problems include modifications that may be made for students with varying skills, abilities, and learning styles. Multiple solutions and multiple methods of solution are encouraged in a classroom that fosters a problem-solving atmosphere. The challenges of creating good problems and maintaining a problem-solving atmosphere in the classroom will be addressed in the next section. In this section we focus on the process and strategies of problem solving.

Polya's Four-Step Process

Probably the most famous approach to problem solving is Polya's four-step process described below (Polya, 1945). Polya identifies the four principles as follows:

1. Understand the problem
2. Devise a plan
3. Carry out the plan
4. Look back

The problem-solving process is merely a general guide of how to proceed in solving problems. In many cases, steps of the process will overlap, thus it may not be possible to perform each step of the process in the order given above. These four principles appear in many elementary-level textbook series as early as the kindergarten grade level.

Use of the problem-solving process and specific strategies may be mentioned in a state or district model or framework.

After describing each of the four steps of Polya's problem-solving process in more detail below, we will discuss a fifth step. This is not suggesting that Polya's process is incomplete. In fact, the fifth step, extend the problem, is mentioned in Polya's manuscript as part of the fourth step—look back. These ideas are separated so that the process of extending the problem, especially relevant for teachers,

does not become lost in the process of verifying the solution.

Step 1: Understand the problem

To correctly solve a problem, you must first understand the problem. Below are some questions that may help lead you (or a student) to an understanding of a given problem. Not every question will be appropriate for every problem.

What are you asked to find or show?

What type of answer do you expect?

What units will be used in the answer?

Can you give an estimate?

What information is given? Do you understand all the terms and conditions?

Are there any assumptions that need to be made or special conditions to be met?

Is there enough information given? If not, what information is needed?

Is there any extra information given? If so, what information is not needed?

Can you restate the problem in your own words?

Can you act out the problem?

Can you draw a picture, a diagram, or an illustration?

Can you calculate specific numerical instances that illustrate the problem?

Step 2: Devise a plan

Use problem-solving strategies. The “plan” used to solve a problem is often called a problem-solving strategy. For some problems, you may begin using one strategy and then realize that the strategy does not fit the given information or is not leading toward the desired solution; in this case, you must choose another strategy. In other cases you may need to use a combination of strategies. Several problem-solving strategies are described below (in no particular order). All of these strategies can be found in elementary-level mathematics textbooks at around the third- or fourth-grade level; many of these strategies are introduced as early as the prekindergarten level.

- **Use guess and check.** When a problem calls for a numerical answer, a student may make a random guess and then check the guess with the facts and information given within the problem. If the guess is incorrect, the student may make and check a new guess. Each subsequent guess should provide more insight into the problem and lead to a more appropriate guess. In some instances the guess and check strategy may also be used with problems for which the answer is non-numerical.
- **Draw a picture or a diagram/use a graph or number line.** A picture or graph may illustrate relationships between given facts and information that are not as easily seen in word or numerical form.

- **Use manipulatives or a model/act it out.** When a problem requires that elements be moved or rearranged, a physical model can be used to illustrate the solution.
- **Make a list or table.** A list or table may be helpful to organize the given information. It may be possible to make an orderly list or table of all possible solutions and then to choose the solution that best fits the given facts and information from this list. In some problems, the answer to the problem is a list or table of all possible solutions.
- **Eliminate possibilities.** When there is more than one possible solution to a problem, each possibility must be examined. Potential solutions that do not work are discarded from the list of possible solutions until an appropriate answer is determined.
- **Use cases.** It is possible to divide some problems into cases. Each case may be separately considered.
- **Solve an equivalent problem.** In some instances it is easier to solve a related or equivalent problem than it is to solve a given problem.
- **Solve a simpler problem.** It may be possible to formulate and solve a simpler problem than the given problem. The process used in the solution of the simpler problem can give insight into the more complex given problem.
- **Look for a pattern.** Patterns are useful in many problem-solving situations. This strategy will be especially useful in solving many real-world problems. "Patterns are a way for young students to recognize order and to organize their world" (NCTM, 2000, p. 91).
- **Choose the operation/write a formula or number sentence.** Some problems are easily solved with the application of a known formula or number sentence. The difficulty often lies in choosing the appropriate formula or operation.
- **Make a prediction/use estimation.** One must closely consider all elements of a problem in order to make a prediction or use estimation. This careful consideration may provide useful insight into the problem solution.
- **Work the problem backward.** If the problem involves a sequence of steps that can be reversed, it may be useful to work the problem backward. Children at the early childhood level may already have some experience in working backward. In solving many mazes and puzzles, it is sometimes easier to begin at the end than to begin at the beginning.
- **Use logical reasoning.** Mathematics can and should make sense. Logical reasoning and careful consideration are sometimes all that is required to solve a mathematics problem.

Step 3: Carry out the plan

Once a problem has been carefully analyzed and a plan is devised, if the plan is a suitable one for the given problem, it is usually a relatively simple process to carry out the plan. However, in some cases the original plan does not succeed and another plan must be devised. The original strategy may need to be modified, or a new strategy may be selected. Students must realize that not every problem will be solved within the first attempt. A failed attempt can be viewed as a learning experience. Try to help students avoid getting frustrated or discouraged. Cooperative learning teams can be used to encourage and engage students. Computers, calculators, or other manipulatives may be useful tools

when routine tasks are involved.

Step 4: Look back

Once an answer or solution is found, it is important to check that solution. Check all steps and calculations within the solution process. Below are some questions that you (or your students) may find useful in the looking back process.

Is the answer reasonable?

Is there another method of solution that will easily verify the answer?

Does the answer fit the problem data?

Does the answer fulfill all conditions or requirements of the problem?

Is there more than one answer?

Will the solution process used be valuable in solving similar or related problems?

Step 5: Extend the problem

For a classroom teacher, an important part of the problem-solving process should involve trying to create similar or related problems. A given problem may need to be simplified in order to be used at a specific classroom level or with students that have special needs. A teacher may wish to make a problem more complicated or to create similar related problems that are more difficult. Elementary school students often extend the problem as part of a journal writing exercise as they write their own story problems for a given situation.

It may be possible to generalize specific instances of a given problem. Teachers must be on the lookout for opportunities to have students generalize and make conjectures. Teachers should look for connections that can be made between given mathematics problems and solutions and real-life situations. Teachers should also look for connections between given mathematics problems and their solutions and other subject areas. There are many excellent articles that deal specifically with posing problems and extending textbook exercises. See, for example, the articles by Butts; Barnett, Sowder, and Vos; LeBlanc, Proudfit, and Putt; and Silver and Smith in the NCTM 1980 Yearbook: Problem Solving in School Mathematics.

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